Comput. Math. Applic. Vol. 14, No. 5, pp. 305–423, 1987
Printed in Great Britain.

0097-4943/87 $3.00 + 0.00

CHAPTER 1

INTRODUCTION

Ocean acoustic wave propagation problems deal with the solution of representative partial differential equations. These equations, which govern realistic physical ocean acoustic phenomena, are all regarded as *wave equations*. Because of the complex nature of the ocean, the various wave equations can be very complicated in nature and permit a closed form solution only in very simple cases. This motivated ocean acousticians as well as ocean scientists and engineers to consider specialized approximations to these problems for simplicity. Thus, a number of wave equations are in existence in different forms; each has its own advantages due to special approximations or treatments. The approximations made generally result in a loss of accuracy or limit the range of validity of the solution. Even when these approximations permit a closed form solution; it usually involves special functions, integrals, etc., which are often not convenient to evaluate. In such situations, direct numerical solution of the wave equation may offer significant advantages, not only by permitting treatment of more realistic environments but also from a computational viewpoint. This monograph demonstrates the applicability of numerical methods for the efficient solution of complex ocean acoustic wave propagation problems.

Numerical methods which have useful applications to ocean acoustic wave propagation problems did not receive much attention or interest until numerical ordinary differential equation methods as well as finite difference schemes were introduced for solving these problems. Due to the rapid growth of supercomputers and modern numerical techniques, solutions for complicated scientific problems are now possible. Ocean scientists have, however, not yet taken full advantage of modern numerical techniques and advanced supercomputer technology. Among these numerical methods the finite difference is well-known, straightforward and stands out as being universally applicable to most scientific problems because of its generality and unconditional stability. Moreover, it is not difficult to implement into computer codes. However, the formulation of efficient finite difference schemes is an art; the precise formulation determines the size of the computational grid, and thus affects the computation speed.

The monograph begins with a description of general ocean acoustic wave propagation problems which are mathematically represented by a partial differential equation, the *wave equation*. Appropriate initial and boundary conditions need to be prescribed for the problem to be well-posed. The wave equation, conventionally regarded as the reduced wave equation, is a scalar elliptic equation whose solution consists of transmitted and reflected fields in three dimensions. It is convenient to express the wave equation in cylindrical coordinates in a three-dimensional ocean, since in most cases the wave field has a very small azimuthal angular dependence. This reduces the three-dimensional problem to a two-dimensional one which is a little easier to handle. Thus, a solution of the two-dimensional wave equation is sought which still consists of a transmitted field and a reflected field. A class of ocean acoustic propagation problems which typically fall into the two-dimensional category is long range propagation at low frequencies. This class of problems can best be represented by a pseudopartial differential equation with complex coefficients. In the second chapter, "Ocean acoustic wave propagation problems", we derive this type of pseudopartial differential equation and discuss how this equation reduces to a parabolic wave equation of the Schrödinger type. Approaches that yield both transmitted and reflected fields using an operator splitting technique are discussed. Due to the nature of this class of long range, low-frequency propagation problems, we generally require only the solution for the transmitted field. The representative pseudopartial differential wave equation sets the stage for us to apply numerical finite difference methods to obtain the transmitted field by a marching process.

The third chapter, "Finite difference schemes", is devoted to fully describing the development of an implicit finite difference scheme applicable to the problem addressed. Finite difference techniques are described in detail from the development of fundamental concepts to the analysis of discretization errors in a style particularly suitable for readers who have little knowledge of finite difference concepts. To solve the representative wave equation, both explicit and implicit finite difference schemes are developed. When the conventional explicit schemes is used to solve the parabolic equation with real coefficients, e.g. the Euler scheme, it is conditionally stable. We show that if the same conventional explicit technique is applied to solve the parabolic wave equation with imaginary coefficients, the resulting computational scheme is unconditionally unstable. This provides a strong motivation for applying an implicit scheme to solve the parabolic wave equation. The technique that we develop is an implicit finite difference scheme of the Crank–Nicolson type which will be referred to as the IFD scheme. The basic formulation of this scheme and the theory regarding its consistency, stability and convergence are described in detail.

Implementation of the finite difference scheme to solve the wave equation, requires the application of appropriate initial and boundary conditions. Boundary conditions for the reduced elliptic wave equation involve not only both surface and bottom boundaries, but also the "wall" boundary that restricts the solution to the computational range of interest: solutions of the elliptic wave equation are solutions of a pure boundary value problem. The special class of problems discussed in the previous chapter can be solved by a marching process and do not require a wall boundary condition, thus, a pure boundary value problem is transformed into an initial boundary value problem. However, a different kind of boundary condition known as the interface boundary occurs and needs to be treated realistically and accurately. In the ocean, temperature, salinity and pressure affect the sound speed as well as the density structure of the water mass creating a layered medium. The sediments that compose the ocean floor are layered due to periodic deposition of sedimentary material. These phenomena stratify the ocean environment into a layered medium, thus, forming interfaces. At each interface, "continuity conditions" must hold, i.e. the pressure and normal component of particle velocity are continuous at the interface. This type of interface can be handled elegantly by a finite difference technique which is new to ocean acoustics. Numerical treatment of the interface conditions by the finite difference technique not only produces accurate, realistic results but also advances the state-of-the-art in numerical solution of ocean acoustic wave propagation problems. The treatment of interface conditions by the finite difference technique is a major portion of Chapter 4, "Initial and boundary conditions". An extended treatment to the case of irregular interfaces is also presented. Although the mathematics involved is very complicated, the procedure is clear and is not difficult to follow. To demonstrate the effects of interface conditions we include two benchmark test problems to show the validity of the numerical treatment. The selection of the time step size for the heat equation presents no problem for the Crank–Nicolson scheme but it presents difficulty for the wave equation of the Schrödinger type. Following Chapter 4, is an analysis of range step size selection.

Long range acoustic propagation in the ocean is dominated by energy that propagates over a narrow angular regime with respect to the horizontal direction. This occurs because energy at high angles interacts strongly with the boundaries and is rapidly attenuated. The parabolic wave equation we discussed in an earlier chapter is an equation of the Schrödinger type which can accommodate only a narrow angle of propagation; for this reason the equation which we introduced as the "standard" parabolic wave equation is also called the "narrow angle" parabolic equation (PE). For many cases, however, the narrow angle PE is not accurate enough to represent the wave field. To accommodate wide angle propagation, wide angle wave equations have been developed which, mathematically speaking, are again pseudopartial differential equations. Chapter 6 is devoted to the fundamental mathematical development of the wide angle wave equation and its solution. The basic questions which are addressed in this chapter are the manner in which the development accommodates wide angle propagation and how the propagation angle is measured. The key to this development is the derivation of a satisfactory mathematical representation of a square root operator which we choose to represent by a rational function approximation to the desired order of accuracy. In this chapter, rational function approximations are derived to handle angles of propagation in two different ranges: 10° and 40° angular range.

In this monograph we show that the IFD scheme is an effective solution to the parabolic wave equation. In fact, there are a number of other solutions to the parabolic wave equation as well which are worthwhile studying. Among these methods is a special class of explicit finite difference schemes. Remember that the motivation for applying the IFD schemes to solve the parabolic wave equation was its advantageous unconditional stability over conventional explicit schemes. We have already proved the instability of conventional explicit schemes when used to solve equations of the Schrödinger type. However, it is known that explicit finite difference schemes have a number of desirable advantages: they are easy to implement and require less storage. These advantages are specially desirable for solving multi-dimensional problems. In addition, it is often easy to vectorize an explicit scheme on the many pipeline-oriented computers available today. Then, it is natural to ask the question whether there exist alternative stable explicit schemes for the Schrödinger equation. This question has been answered affirmatively for a case where an appropriate dissipative term was introduced into an explicit scheme which was conditionally stable. A class of new explicit schemes have been developed based on this approach. We include these new explicit schemes in Chapter 7, "Applicable solution methods other than the implicit finite difference scheme". Formulations as well as theoretical developments are included with proofs. These developments represent a significant advance in numerical techniques. These new explicit schemes can also be used effectively to solve problems in other areas of physics.

We have developed representative wave equations for ocean acoustics, and introduced an efficient numerical implicit finite difference method to solve these wave equations. In addition, we developed numerical procedures to treat realistic ocean environmental effects. To combine all these effectively together, we constructed a computer code, the IFD computer code. We have made this code as reliable as possible, easy to use, accurate, general purpose and easy to modify. Not only can this code be used for research purposes, but it can also be used to make production runs. At the date of publication of this monograph, there are a large number of satisfied IFD code users among universities and research laboratories nationwide as well as worldwide. For the IFD code user, we present a chapter entitled "Representative test examples". Most of them are benchmark test problems. Users can exercise these problems to gain experience in solving realistic problems and also to get a "feel" for the computer code. We have carefully selected these test examples for the sole purpose of exhibiting different ocean environmental effects, to examine the computational accuracy, and to demonstrate the versatility of the model. It should be pointed out that this IFD model is not limited to only the solution of the class of ocean acoustic wave propagation problems addressed earlier in this monograph. It also provides practical applications to other scientific and engineering problems frequently arising in the fields of atmospheric acoustics, plasma physics, quantum mechanics, optics, seismology, electromagnetics, etc.

This monograph concludes with a comprehensive listing of the IFD computer code designed in FORTRAN language. This code represents a self-contained efficient numerical treatment of a class of computational ocean acoustic wave propagation problems.